Design of risk sharing for variable annuities

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INNOVATING ACTUARIAL RESEARCH ON FINANCIAL RISK AND ERM
Outline

1. Introduction

2. Recall on group self-annuitization (GSA)

3. On the risk-sharing GSA

4. Numerical results

5. Conclusion and perspectives
Introduction

▶ Motivation: In the recent years, the pension reform and longevity risk have weakened the annuity market. The revival of annuity market will come from the design of new risk-sharing annuities.

There exists several annuities for which the whole risk is borne by the insurer. Researchers are paying attention on the risk-sharing annuities. Some new designed annuities are pooled annuities where the whole risk is borne by the pool (i.e by the annuitants); e.g the group self-annuitization (GSA).

GSA schemes operate like conventional annuities for which uncertain future mortality risks are shared in the pool.
Introduction

▶ Literature :
▶ Piggott et al. (2005 [2]) proposed a formal analysis of benefit adjustment from a longevity risk-pooling fund : the GSA

▶ Qiao & Sherris (2013 [1]) have extended [2] by showing the extent to which the pooling mechanism can be made effective

Our goal : Propose some (partial or whole) risk-sharing GSA (i.e risk sharing between the pool and the insurer)
Framework

- Filtered probability space \((\Omega, \mathcal{F}_t, \mathbb{F}, \mathbb{P})\) made of a risky \(S_t\) and a risk-free \(B_t\) assets

- Strategy: Constant proportion \(x \in [0, 1]\) is invested on \(S_t\) and proportion \((1 - x)\) on \(B_t\); with a dynamic rebalancing. Investment on \([t, T]\) is

\[
A(t, T) = A(0, t) \ e^{(\mu x + (1-x)r + \sigma^2 x^2/2)(T-t) + x \sigma W_{T-t}},
\]

with \(A_t = A(0, t); \ r, \ \sigma, \ \mu \in \mathbb{R}_+^*\)

- Mortality model: force of mortality follows the Hull-White (HW) model

\[
d\mu^x_t = (\theta(t) - a \mu^x_t)dt + \sigma^\mu dW^\mu_t, \quad \text{for all } t \geq 0
\]

where \(\theta(t) = Ae^{Bt}\) (Gompertz model).
Assumptions

- $W_t$ independent of $W_t^\mu$;
- Consider an open (homogeneous or heterogeneous) pool of policyholders;
- Only longevity and equity risks are considered;

Features of GSA:

- Longevity and equity risks fully borne by the pool;
- Annual benefits defined iteratively.
Recall on GSA – Notation

- $B_t$ : total benefit of the pool at time $t$;
- $k_xB_{i,t}$ : benefit at time $t$ of the $i^{th}$ annuitant entering the pool at age $x$, $k$ period of time ago;
- $F_t$ : total fund of the pool at time $t$;
- $k_x\hat{F}_{i,t}$ : fund value at $t$ of the $i^{th}$ policyholder including the inheritance from those who died during the period $[t - 1, t]$;
- $k_xF_{i,t}$ : fund value at time $t$ of the $i^{th}$ policyholder who entered the pool at age $x$, $k$ period ago;
- $spx_{x,t}$ : the survival probability at time $t$ of an annuitant aged $x$ of living $s$ more years;
- $\bar{a}_x(t)$ : annuity factor at time $t$ of a policyholder aged $x$. 

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Recall on GSA

Proposed in 2005 by Piggott et al. [2]

- Capture the mortality changes using a mortality experience adjustment factor:

\[
\text{MEA}_t = \frac{F_t}{\sum_{k \geq 1} \sum_{x} p_{x+k-1,t-1} \sum_{x} \sum_{A_t}^k x F_{i,t}};
\]

- Capture the updates on the mortality information available at each time by a change expectation adjustment factor:

\[
\text{CEA}_t = \frac{\ddot{a}_{x+k}(t-1)}{\ddot{a}_{x+k}(t)};
\]

- Capture the investment changes using an interest rate adjustment factor:

\[
\text{IRA}_t = \frac{1 + R_t}{1 + R}
\]
Recall on GSA

- Individual benefit at entrance:

\[ 0_x B_{i,t} = 0_x F_{i,t} \frac{\tilde{a}_x(t)}{\bar{a}_x(t)}. \]  

- Individual benefit afterwards i.e. \( k > 0 \):

\[ k_x B_{i,t} = k^{-1} B_{i,t-1} \text{ MEA}_t \text{ IRA}_t \text{ CEA}_t; \]
Recall on revised GSA

Proposed in 2013 by Qiao & Sherris ([1])

- Increase the effectiveness of the pooling mechanism
- Capture the mortality changes using a single mortality adjustment factor:

\[
\text{TEA}_t = \frac{F_t}{\sum_k \sum_{x} p_{x+k-1,t-1} \frac{\ddot{a}_{x+k}(t)}{\ddot{a}_{x+k}(t-1)} \sum_x k F_{i,t}};
\]

- \( \text{TEA}_t \) improve the dependence of mortality changes across the pools
- Individual benefit for \( k > 0 \):

\[
k^x B_{i,t} = k^{-1} x B_{i,t-1} \text{TEA}_t \text{ IRA}_t;
\]
On the risk-sharing GSA

- Shifting a constant or variable proportion of the whole and/or partial risk to the insurer;
- Then reduction of the risk borne by the pool and increase of annual benefits;
- $k \frac{R_{i,t}}{x}$ denotes the individual annual benefit of the risk-sharing (revised) GSA;
- Lower bound threshold sharing: individual benefit at least equal to the threshold $\frac{x}{R_{i}}$;
- Direct or static sharing: the risk is statically shared between the insurer and the pool.
Lower bound threshold sharing

- $\gamma_t \in [0, 1]$ is the proportion of risk shifted to the insurer;

- Annual lower bound: $xR_i(\gamma_t) = \gamma_t^0 xR_{i,t}$

- Defined the individual annual benefit:

  $$\overline{kxR_{i,t}}(\gamma_t) = \max \left( \frac{k}{x} R_{i,t}, \overline{xR_i}(\gamma_t) \right) ;$$

  where $\frac{k}{x} R_{i,t}$ is defined by a GSA or equals to

  $$\frac{k}{x} R_{i,t} = \frac{k-1}{x} R_{i,t-1}(\gamma_{t-1}) \text{ Adjust}_t.$$
Static sharing

- $\beta'_t, \beta_t \in [0, 1]$ are respectively the proportion of mortality and equity risks shifted to the pool;

- Defined the individual annual benefit of the risk-sharing GSA:

$$\overline{k_xB_{i,t}}(\beta_t, \beta'_t) = \overline{k^{-1}_xB_{i,t-1}(\beta_t, \beta'_t)} [\beta_t \text{MEA}_t \text{CEA}_t + (1 - \beta_t)] \times [\beta'_t \text{IRA}_t + (1 - \beta'_t)];$$

- Individual annual benefit of the risk-sharing revised GSA

$$\overline{k_xB_{i,t}}(\beta_t, \beta'_t) = \overline{k^{-1}_xB_{i,t-1}(\beta_t, \beta'_t)} [\beta_t \text{TEA}_t + (1 - \beta_t)] \times [\beta'_t \text{IRA}_t + (1 - \beta'_t)];$$
Numerical parameters

- Consider a $n = 9$ years term annuity with $x = 15\%$, $r = R = 1\%$ and a closed heterogeneous pool:

<table>
<thead>
<tr>
<th>Group $i$</th>
<th>Individual premium $F_{i,0}$</th>
<th>Initial age $x_i$</th>
<th>Number of annuitants $N_{i,0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1000€</td>
<td>65</td>
<td>100</td>
</tr>
<tr>
<td>G2</td>
<td>700€</td>
<td>70</td>
<td>700</td>
</tr>
</tbody>
</table>

- $\mu = 5.8702\%$; $\sigma = 20.4172\%$ : calibrated from the S&P500 indexes using MLE

- Consider a unisex mortality table of individuals initially aged 65 and 70 (source: IA|BE mortality tables 2015)

- Calibration of HW model : MSE

- Simulation approach : MC
Results: Static sharing

Group 1

Group 2
Results: Lower bound threshold sharing

Group 1

Annual benefit of Group 1: Term annuity (MEA)

Group 2

Annual benefit of Group 2: Term annuity (MEA)
Results

Table 1: Annuitants’ annual benefit at $t = 7$ for a $n = 9$ years term annuity; $\gamma = 1 - \beta$ and $\beta = \beta'$. 

<table>
<thead>
<tr>
<th></th>
<th>Group1</th>
<th></th>
<th></th>
<th>Group2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static ($\beta$)</td>
<td>Threshold ($\gamma$)</td>
<td>Static ($\beta$)</td>
<td>Threshold ($\gamma$)</td>
<td></td>
</tr>
<tr>
<td>$\beta = 5%$</td>
<td>110.6</td>
<td>105.9</td>
<td>79.1</td>
<td>75.8</td>
<td></td>
</tr>
<tr>
<td>$\beta = 50%$</td>
<td>102.5</td>
<td>94.4</td>
<td>73.3</td>
<td>67.5</td>
<td></td>
</tr>
<tr>
<td>$\beta = 95%$</td>
<td>94.9</td>
<td>94.4</td>
<td>67.8</td>
<td>67.5</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion and perspectives

A range of annuities can be built from the risk-sharing GSA: we have

(i) Conventional annuity if no risk borne by the pool
(ii) GSA or revised GSA if no risk borne by the insurer
(iii) Intermediate cases if the risk of the insurer is in \((0, 1)\)

SO...?? Find \(R(\beta'_t, x)\) and \(sp_{x_k}(\beta_t)\) such that

\[
\ddot{a}_{x_k}(t, \beta_t, \beta'_t) = \sum_{s=0}^{\infty} \left( \frac{1}{1 + R(\beta'_t, x)} \right)^s sp_{x_k}(\beta_t)
\]
Conclusion and perspectives

Question: Up to which extend could this be reasonable enough for an insurer?

Answers (Future work!):
⇒ Find the risk proportions that minimise the solvency capital (SC) of the insurer.
⇒ Compute the insurer’s risk premium for a given risk proportion

What next...
▷ Design and price successive annuities using risk-sharing annuity.


THANK YOU FOR YOUR KIND ATTENTION!