Frailty modelling in a multi-state framework

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Table of Contents

Introduction and Motivations for the study

Life and health insurance products include heterogeneity of the persons with respect to

- ✓ probability of disablement
- ✓ probability of entering long-term case states
- ✓ mortality of active people
- ✓ mortality of disabled people
- ✓ probability of recovery and returning again in the health state

Introduction and Motivations for the study

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  - individual-specific factors (genetic features, lifestyle, attitude towards health)
  - Not embedded into underwriting process

Introduction and Motivations for the study

Modelling heterogeneity

- Observable risk factors
  - Individual valuation approaches - additive or multiplicative adjustments to the average disability rate are applied in the pricing procedure.
- Unobservable risk factors
  - Collective valuation models - not relevant in life insurance. Long term and multi-year characteristic of the life insurance contracts, dependency of annual disability rates at the attained age underlying difficulties in depicting the aspects that cannot be observed indicate the complexity of dealing with the unobservable heterogeneity

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  - However unobservable heterogeneity can be quantified by adopting the concept of the individual frailty (Beard 1959) and (Vaupel, Manton, and Stallard 1979).

Introduction and Motivations for the study

Our aim: Composite a series of multi-year models in Markov time-discrete framework with three states (active-disabled-dead), starting from the basic binomial assumption, and forward-looking into models holding various stochastic aspects caused by:

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- uncertainty in the assigned disability probability;
- observable heterogeneity, with potential random group sizes;
- unobservable heterogeneity quantified by frailty model.

Introduction and Motivations for the study

Model output results represents the main portfolio characteristic in terms of:

- expected value of the benefit paid in case of dead;
- variance of the benefit paid in case of dead.

Application with respect to the disability probability finds its contribution into health insurance:

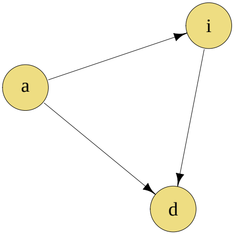
- providing a lump sum in case of Disability Insurance, or Personal Accident;
- providing Long-term care annuities benefits

Introduction and Motivations for the study

Determing as $\mathcal{L}$ the infinite state space, disability insurance cover the state space is represented as: $\mathcal{L} = \{\mathrm{a}, \mathrm{i}, \mathrm{d}\}$, where $\mathcal{L}$ is subset of the set of pairs (i, j) such that:

$$\mathcal{T} \subseteq \{(\mathrm{i}, \mathrm{j}) |\ \mathrm{i} \neq \mathrm{j}; \quad \mathrm{i}, \mathrm{j} \in \mathcal{L}\}, \quad \text{set of direct transitions.}$$

Our multi-state disability insurance model $(\mathcal{L}, \mathcal{T})$,
$\mathcal{T} = \{(\mathrm{a}, \mathrm{i}), (\mathrm{a}, \mathrm{d}), (\mathrm{i}, \mathrm{d})\}$ is depict in the following direct graph:



Introduction and Motivations for the study

Basic key guide provided by:

> Pollard, A. H. (1970). Random mortality fluctuations and the binomial hypothesis. Journal of the Institute of Actuaries, 96: pages 251 - 264

Some models are formalized with numerical evaluation by M. Valente (Master thesis, University of Trieste, March 2018)

Table of Contents

Frailty Model

Frailty model is a stochastic approach, based on non-negative real-valued variable called frailty, which level expressed the unobservable risk factors affecting the individual disability, mortality, recovery, etc. The fundamental conclusion is that the people with higher level of frailty tend to get on average earlier disablement compared to the others.

The first definition about the non-negative quantity and the frailty approach is referred to (Beard 1959) but formally represented by (Vaupel et al., 1979)

Frailty Model

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- As regard to the relation between the frailty and the individual age:

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  - discrete models - individual frailty takes a finite number set of specific possible values;
  - continuous models - the individual frailty takes an interval of specific possible values.
- As regard to the relation between the frailty and the individual age:
  - constant frailty model (Fixed frailty approach) the special values of the individual frailty level is unknown, but does not change with the lifetime;
  - variable frailty model, the individual frailty level is age-dependent, stochastically change over the whole life span.

Frailty Model

Model choice - depends on what risk factors will be summarized by the frailty model.

In our study - continuous frailty model represent the individual random probability of disablement.
Random variable W with four-parameter beta distribution $\mathrm{Beta}(\alpha, \beta, \mathrm{a}, \mathrm{b})$, such that $\mathrm{a} = 0$, $\mathrm{b} < 1$.

$$\begin{aligned} \mathrm{E(W)} &= \mathrm{b}\frac{\alpha}{\alpha+\beta} \\ \mathrm{Var(W)} &= \mathrm{b}^2\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \end{aligned}$$

Table of Contents

From deterministic to frailty modelling

Defining the portfolio

- N_0 individual policies;
- homogeneous portfolio with respect to the age and policy term;
- portfolio close to new members;
- random number of events (disablement);
- lump sum benefit D in case dead occurs (The dead captures the two previous state of the insured), thus the occurrence of disability event should be analysis.

Focus on:

- Expected value of the result of interest E(D);
- Variance of the result of interest Var(D).

From deterministic to frailty modelling

Several cases are studied

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- Deterministic case (simplest binomial case);
- Possible risk classes are considered;
- Possible uncertainty is allowed for;
- Possible for real-valued individual frailty assumptions are adopted.

From deterministic to frailty modelling

- Deterministic case
  - N_0 individual risks;
  - probability of disablement: p_x^{ai} is a given value
  - in case of grouping allowed
    - group size

$$N_1, N_2, \dots, N_r, \qquad \text{as} \quad \sum_{j=1}^{r} N_j = N_0$$

    - assigned probabilities

$$p_x^{ai(1)} < p_x^{ai(2)} < \dots < p_x^{ai(r)}, \quad \overline{p_x^{ai}} = \sum_{j=1}^{r} \frac{N_j}{N_0} p_x^{ai(j)} = p_x^{ai}$$

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From deterministic to frailty modelling

- Possible risk classes are considered
  - risk classes with given sizes;
  - risk classes with random sizes.

- Possible uncertainty is allowed for $\mathrm{p}_{\mathrm{x}}^{\mathrm{ai}}$ as a general common random value W, Beta distributed :
  - the same for the whole insured persons (No grouping);
  - allowed for more risk classes due to observable risk factors (Grouping) $\mathrm{W}_1, \mathrm{W}_2, \ldots, \mathrm{W}_\mathrm{r}$, with increasing expected values.

From deterministic to frailty modelling

- Possible for real-valued individual frailty assumptions
  - random individual value $W^{(j)}$ (No grouping, Continuous Frailty modelling), Beta distribution, same parameters for all individuals;
  - random individual value, Beta distribution,(Grouping, Discrete Frailty modelling) $W_k^{(j)}$ parameters depend on the group k, $k = 1, \dots s$. Same parameters for Beta distribution for all individuals inside the group. Group sizes unknown.

From deterministic to frailty modelling

- Homogeneity
- No uncertainty

Case 1
N_0 individuals
1 group
p_x^{ai}

From deterministic to frailty modelling

- Observable heterogeneity
- Deterministic group sizes
- No uncertainty

Case 2
N_0 individuals, r groups
$N_1, N_2, \ldots, N_r$ given
$p_x^{ai(1)}, p_x^{ai(2)}, \ldots, p_x^{ai(r)}$ given

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From deterministic to frailty modelling

- Homogeneity
- Uncertainty

Case 3
N_0 individuals, 1 group
p_x^{ai} random Beta(α, β, a, b)

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From deterministic to frailty modelling

- Unobservable heterogeneity
- Continuous frailty modelling

Case 4
$\mathrm{N_0}$ individuals, 1 group
$\mathrm{p_x^{ai(j)}}$, $\mathrm{j} = 1, 2, \ldots, \mathrm{N_0}$ random $\mathrm{Beta}(\alpha, \beta, \mathrm{a}, \mathrm{b})$

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From deterministic to frailty modelling

- Unobservable heterogeneity
- Discrete frailty modelling

Case 5
N_0 individuals, s groups
$N^{(1)}, N^{(2)} \ldots, N^{(s)}$ random
$\underline{N} = (N^{(1)}, N^{(2)} \ldots, N^{(s)})$ has Multinomial $(N_0, f_1, f_2, \ldots, f_s)$
$p_x^{ai(j)}$, $j = 1, 2, \ldots, N_0$ random Beta$(\alpha_k, \beta_k, a_k, b_k)$, $k = 1, \ldots, s$

From deterministic to frailty modelling

- Observable heterogeneity (Deterministic group sizes)
- Unobservable heterogeneity
- Continuous frailty modelling

Case 6
N_0 individuals, r groups
$N^{(1)}, N^{(2)} \ldots, N^{(r)}$ given
$p_x^{ai(j)}$, $j = 1, 2, \ldots, r$ random Beta$(\alpha_j, \beta_j, a_j, b_j)$, $j = 1, \ldots, r$

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Table of Contents

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Numerical results

Data

- $N_0 = 10000$;
- possible grouping $r = 3$
  - $p_x^{ai} = 0.02$, in case of grouping $p_x^{ai(1)} < p_x^{ai(2)} < p_x^{ai(3)}$ with $\overline{p_x}^{ai} = 0.02$.
- Uncertainty and/or frailty
  - In case of no grouping p_x^{ai} is random with $\text{Beta}(2.2, 3.3, 0, 0.05)$;
  - In case of grouping: different Beta distributions, with increasing expected values.

▶ Results of interest is the sum assured for dead active (DA)

Numerical results

Effect of observable risk factors Case1 $\longleftrightarrow$ Case2

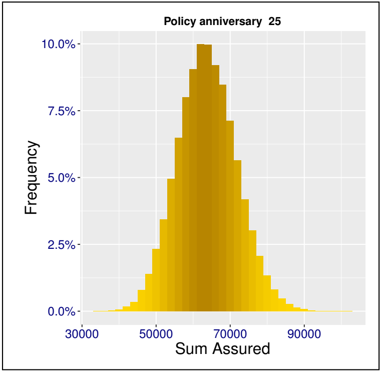

Figure 1: Effect of observable risk factors, Case 1, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

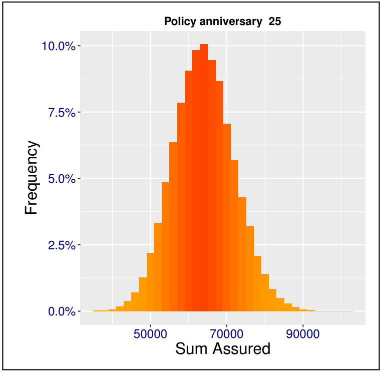

Figure 2: Effect of observable risk factors, Case 2, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

Numerical results

Effect of uncertainty Case1 $\longleftrightarrow$ Case3

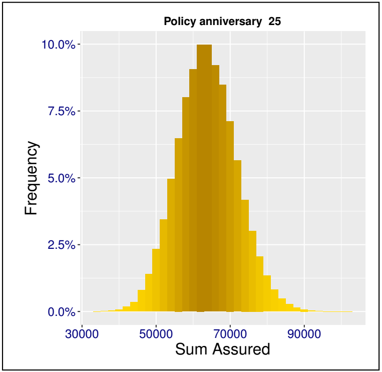

Figure 3: Effect of uncertainty, Case 1, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

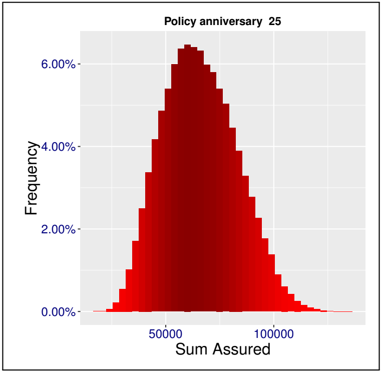

Figure 4: Effect of uncertainty, Case 3, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

Numerical results

Effect of diverse frailty modelling Case5 $\longleftrightarrow$ Case4

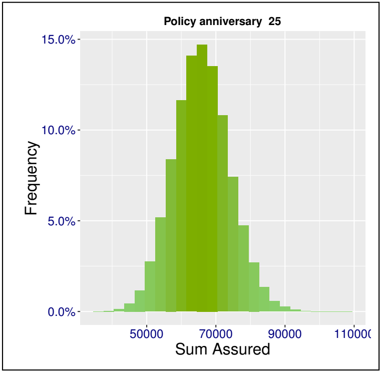

Figure 5: Discrete frailty model, Case 5, Distribution of the sum assured (DA) at time $t = 25$; $\mathrm{Sim}_n = 100000$

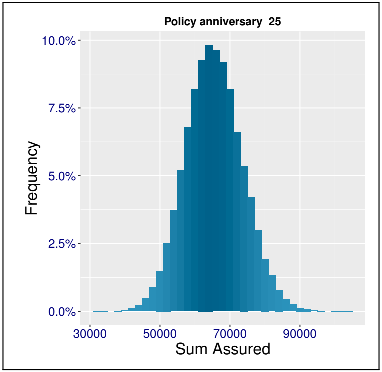

Figure 6: Continuous frailty model, Case 4, Distribution of the sum assured (DA) at time $t = 25$; $\mathrm{Sim}_n = 100000$

Numerical results

Effect of frailty Case1 $\longleftrightarrow$ Case4

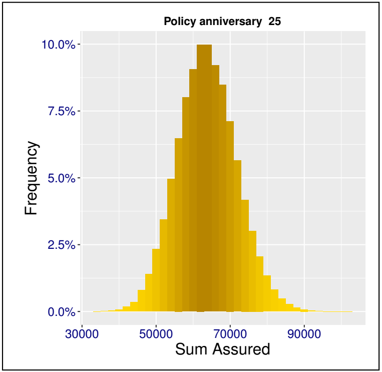

Figure 7: Effect of frailty,Case 1, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

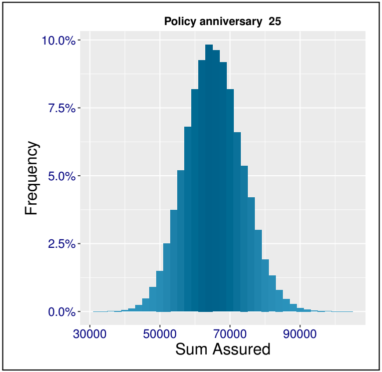

Figure 8: Effect of frailty, Case 4, Distribution of the sum assured (DA) at time t = 25; $\mathrm{Sim_n} = 100000$

Numerical results

Effect of both observable and not observable risk factors
Case2 $\longleftrightarrow$ Case6

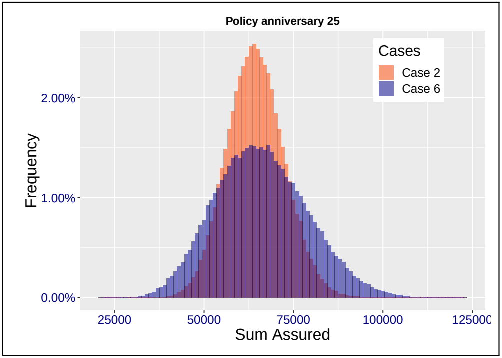

Figure 9: Effect of both observable and not observable risk factors, Case 2 and Case 6, Distribution of the sum assured (DA) at time $t = 25$; $\mathrm{Sim_n} = 100000$

Table of Contents

Conclusion

- Only observable heterogeneity can't improve the portfolio risk profile;

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- Uncertainty and frailty lead to general increase the variance of the result of interest;
- Heterogeneity and grouping lead to construction of several risk classes, contribute to lowering sum assured (DA) variance;
- Frailty explains better the risk profile of the insurance portfolio. Needed in order to explore the impact of the probabilistic structure of the disability rate.

Conclusion

Possible generalizations

- Different benefit amounts;
- Allowing for multiple decrement (competing risks), e.g. mortality and disablement, with different benefit definitions.

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Thank you